Estimating Bias Uncertainty

Howard Castrup, Ph.D.
Integrated Sciences Group
14608 Casitas Canyon Road
Bakersfield, CA 93306
hcastrup@isgmax.com

Abstract - Methods for estimating the uncertainty in the bias of reference parameters or artifacts are presented. The methods are cognizant of the fact that, although such biases persist from measurement to measurement within a given measurement session, they are, nevertheless, random variables that follow statistical distributions. Accordingly, the standard uncertainty due to measurement bias can be estimated by equating it with the standard deviation of the bias distribution. Since the measurement bias of a reference is a dynamic quantity, subject to change over the calibration interval, both uncertainty growth and parameter interval analysis are also discussed.

INTRODUCTION

Measurements are accompanied by measurement errors. The uncertainty in the sign and magnitude of a measurement error is called *measurement uncertainty*. To put this statement in mathematical terms, errors that occur in measurement are random variables that follow statistical distributions. The uncertainty due to a specific error is equal to the standard deviation of the error distribution.[1]

The total error in a measurement is comprised of errors from several possible sources. Among these are parameter error, random error, resolution error, operator bias, sampling error, environmental error, etc. Each error follows a statistical distribution with a standard deviation. Accordingly, there is an uncertainty associated with each error source. The total uncertainty in a measurement is composed of these uncertainties.

Parameter Error

A device parameter can be thought of as a discrete function that is assigned a tolerance specification. For single-parameter devices, such as gage blocks, the parameter specification is synonymous with the device specification. For multi-parameter devices, each parameter has its own function and specification. For example, the 10 vDC function of a multimeter constitutes a parameter. With regard to functions that are specified over a range of values, it should be noted that, for purposes of uncertainty analysis, each measured point within the range constitutes a separate parameter.

Parameter error is a deviation of a device-sensed or device-generated value from an underlying true value. To elaborate, if the device parameter measures the value of a quantity, the parameter error is the difference between the true value of the quantity and the value sensed by the parameter. If the device provides an output and has a nominal or reading value, the parameter error is the difference between the true value of the output and the nominal or reading value.

---

[1] The standard deviation should not be confused with the "expanded uncertainty," which is equal to the standard deviation multiplied by a constant referred to as a "coverage factor." In some circles, it is customary to employ an arbitrary fixed number (e.g., 2 or 3) for this quantity. In this paper, coverage factors are t-statistics determined from confidence levels and degrees of freedom.

An example of the first kind of parameter error is the difference between the true value of a gage block and the value measured by a micrometer. An example of the second kind of parameter error is the difference between the true frequency produced by a signal generator and the value that is indicated by the generator dial or readout.

Parameter Bias

Parameter error is composed of random error in the device-sensed or device-generated value and a systematic component called *parameter bias*. Parameter bias is an error component that persists from measurement to measurement during a "measurement session." Parameter bias excludes resolution error, random error, operator bias and other sources of error that are not properties of the parameter.[2]

Estimating the uncertainty due to parameter bias is the primary subject of this article.

Parameter Bias Composition

Parameter bias may include contributions from systematic long-term variations and from random stresses encountered during usage, calibration or storage. Systematic long-term variations are usually referred to as "parameter drift." Changes due to random stresses are referred to as "random variations." Parameter drift may be linear or nonlinear. The rate of change in parameter value due to drift is called the *drift rate*.

The specific bias that a parameter may have at any given time depends on the following variables

1. The initial bias of the parameter following testing or calibration, referred to as the *beginning-of-period* (BOP) bias.
2. The parameter's drift rate and the time elapsed since testing or calibration.
3. The sum of random variations.

Parameter Bias Uncertainty

To reiterate from above, the uncertainty in the bias of a parameter consists of the uncertainty due to drift, the uncertainty due to random variations and the uncertainty due to testing or calibration. If we let t represent the time elapsed since testing or calibration, the error model is

$$\varepsilon_{bias}(t) = \varepsilon_{BOP} + \varepsilon_{drift}(t) + \sum_{i=1}^{n(t)} \varepsilon_{i,r} , \qquad (1)$$

where

$\varepsilon_{drift}(t)$	= parameter bias due to drift,
ε_{ir}	= random variation due to the ith random stress experienced since testing or calibration,
$n(t)$	= number of random variations occurring over time t,
ε_{BOP}	= bias of the parameter due to testing or calibration.

The uncertainty in the bias of a parameter is obtained by taking the statistical variance of Eq. (1)

$$\begin{aligned} u_{bias}^2(t) &= \mathrm{var}\left[\varepsilon_{bias}(t)\right] \\ &= \mathrm{var}\left(\varepsilon_{BOP}\right) + \mathrm{var}\left[\varepsilon_{drift}(t)\right] + \mathrm{var}\left(\sum_{i=1}^{n(t)} \varepsilon_{i,r}\right) \\ &= u_{BOP}^2 + u_{drift}^2(t) + u_r^2(t), \end{aligned} \qquad (2)$$

---

[2] For purposes of discussion, a measurement session is considered to be an activity in which a measurement or sample of measurements is taken under fixed conditions, usually for a period of time measured in seconds, minutes or, at most, hours.

where it is assumed that errors due to drift, random variations and testing or calibration are statistically independent.[3]

Drift Uncertainty

Parameter drift may be represented by a mathematical function of time, characterized by coefficients. For example, if parameter drift is linear with time, the function could be written

$$\varepsilon_{drift}(t) = bt , \qquad (3)$$

where b is a coefficient, and t is the time elapsed since testing or calibration. For the function defined in Eq. (3), the coefficient b is the parameter drift rate.

The uncertainty due to drift is obtained by taking the variance of Eq. (3)

$$u_{drift}^2(t) = t^2 u_b^2 + b^2 u_t^2 . \qquad (4)$$

The uncertainty u_t is normally not a factor, since we either know the elapsed time or are endeavoring to compute bias uncertainty as a function of time, in which case t is a known input variable. The uncertainty u_b is the uncertainty in the parameter's drift rate. In cases where this drift rate has been determined experimentally, what we usually have at hand is a regression estimate for b, together with a regression variance that expresses the uncertainty in b

$$u_b^2 \simeq \mathrm{var}(b) .$$

This topic will be discussed in detail later in the paper.

Compensation for Drift

If the drift of a parameter has been successfully modeled and the coefficients of the model have been obtained by regression or other methods, the parameter value can be corrected by taking the estimated drift into account. While this may reduce the parameter error or bias, the uncertainty in the correction u_{drift} must still be accounted for. For our simple linear model, this uncertainty is estimated by u_b above. More general and complex models will be encountered later.

Random Effects Uncertainty – Process Variation

In addition to parameter variations due to drift, we have parameter variations due to random effects. The random effects variations may be due to responses to usage stress, shipping and handling, energizing, etc. Again, the sign and magnitude of these variations are unknown. The uncertainty in the sign and magnitude of such a random variation constitutes the long-term random uncertainty or process variation.[4]

Combined Random and Drift Uncertainty – An Analogy

Imagine that you are present at a demonstration of Brownian motion in which a bottle of perfume is uncorked, and the times required for the perfume aroma to reach different parts of the room are measured. At time $t = 0$ (when the bottle is uncorked) a given perfume molecule can be said to be contained within a fairly localized neighborhood. As time passes, the probability that the molecule is contained within this neighborhood becomes comparatively small. If the room were of infinite volume, this probability would eventually vanish to zero.

For parameters with no drift characteristic, variations in parameter value due to random effects are analogous to changes in molecule location due to Brownian motion. At $t = 0$ (immediately following testing or calibration), the parameter value can be said to be contained within a neighborhood whose dimensions are set by the uncertainty in

---

[3] In developing the variance due to random error, it is important to bear in mind that n is a random variable. Hence, the uncertainty u_r is defined according to $u_r = u_{\Sigma \varepsilon_{i,r}}$, rather than $u_r^2 = \Sigma u_{\varepsilon_{i,r}}^2$.

[4] This is not the random uncertainty in measurement, which is due to short-term random variations that occur during measurement.

the testing or calibration process. As t increases, the probability of this containment grows smaller. In other words, the uncertainty in the parameter's value increases. This phenomenon is called *uncertainty growth* [1-3].

We now include parameter drift. Suppose that the perfume experiment is conducted in a tunnel, and that there is a constant air flow present. In this case, perfume molecules again exhibit random Brownian motion, but there is also a systematic motion due to the air flow. If we compare perfume aroma arrival times with and without air flow, we can estimate the drift rate, or, equivalently, the rate of air flow. Of course, this estimate will be subject to the uncertainty of the time and distance measurements and of our sensitivity to the perfume scent.

The foregoing random and systematic variations are analogous to those exhibited by device parameters. We sometimes estimate drift based on the performance of an individual device and sometimes obtain the estimate based on the performance of a sample of devices drawn randomly from a population. The latter approach is analogous to the perfume aroma drift estimate in that, in the perfume experiment, we are not measuring the drift of a specific molecule. We are instead measuring the drift of the molecule population.

ANALYSIS OF BIAS UNCERTAINTY

The uncertainty in the bias of a parameter can be estimated as a function of time elapsed since testing or calibration. The estimate may be obtained using variables data or attributes data.

Variables Data Analysis

Variables data obtained by periodic calibration of equipment parameters can be used to estimate bias uncertainty. The scenario is one in which the "as-found" parameter value measured at each calibration is compared to the "as-left" value measured at the preceding calibration. By plotting the difference between as-found and as-left values as a function of the time elapsed between calibrations, we can develop projections of uncertainty growth over time.

One method of analysis that has been found to be useful in this regard is *regression analysis*. With regression analysis, each point that is entered in an analysis has associated with it the uncertainty of the measurement process that produced its value. In addition to the measurement process uncertainties, the results of uncertainty growth analysis will include other uncertainties due to both the regression fit and to random process variation over time. The latter two contributions are estimated as a byproduct of the analysis.

Estimating the Bias Uncertainty

With regression analysis applied to variables data, u_{drift}, u_r and u_{BOP} can be estimated using the regression model

$$y(t) = a + bt, \tag{5}$$

where a and b are coefficients that are estimated, t represents the time elapsed between calibrations, and $y(t)$ represents the difference between as-found and as-left values at time t. With this model, the uncertainty at any given time t is given by the expression

$$u_{bias}^2 = \mathrm{var}(a) + t^2\,\mathrm{var}(b) + 2t\,\mathrm{cov}(a,b)\,. \tag{6}$$

Comparison with the components of u_{bias} in Eq. (2) shows that

$$u_{BOP}^2 = \mathrm{var}(a)\,,$$

$$u_{drift}^2(t) = t^2\,\mathrm{var}(b)\,,$$

and

$$u_r^2(t) = 2t\,\mathrm{cov}(a,b)\,.$$

This is an interesting result. One of the reliability models employed in attributes data calibration interval analysis, to be discussed later, is the *random walk* model. This model assumes that parameter values change randomly with respect to direction (sign) and magnitude. In the course of its derivation, the uncertainty or standard deviation of the parameter distribution, $u_r(t)$, turns out to be proportional to the square root of the time elapsed between calibrations.

So, by virtue of the linear model that we have chosen for variables data analysis, we end up with the same result, i.e., $u_r(t) \sim \sqrt{t}$.

Before moving on to the estimation of regression coefficients *a* and *b*, it should be mentioned that it is important to bear in mind that the variance and covariance terms of u_{bias} include uncertainty contributions from both the regression fit and from the uncertainties in the measured as-found vs. as-left differences that are entered into the analysis. These contributions will become apparent in the discussion on estimating the variances. They are portrayed in Figure 1.

Figure 1 shows the estimated bias drift over time, the uncertainty due to random effects[5] (process variation) and the measuring process uncertainty for each measured deviation. The uncertainty $u_{drift}(t)$ is indicated by the upper and lower 95% regression confidence limits. The random uncertainty $u_r(t)$ is estimated as 0.002061 cm. The measuring process uncertainties are displayed in the form of upper and lower one-sigma error bars around each measured difference.
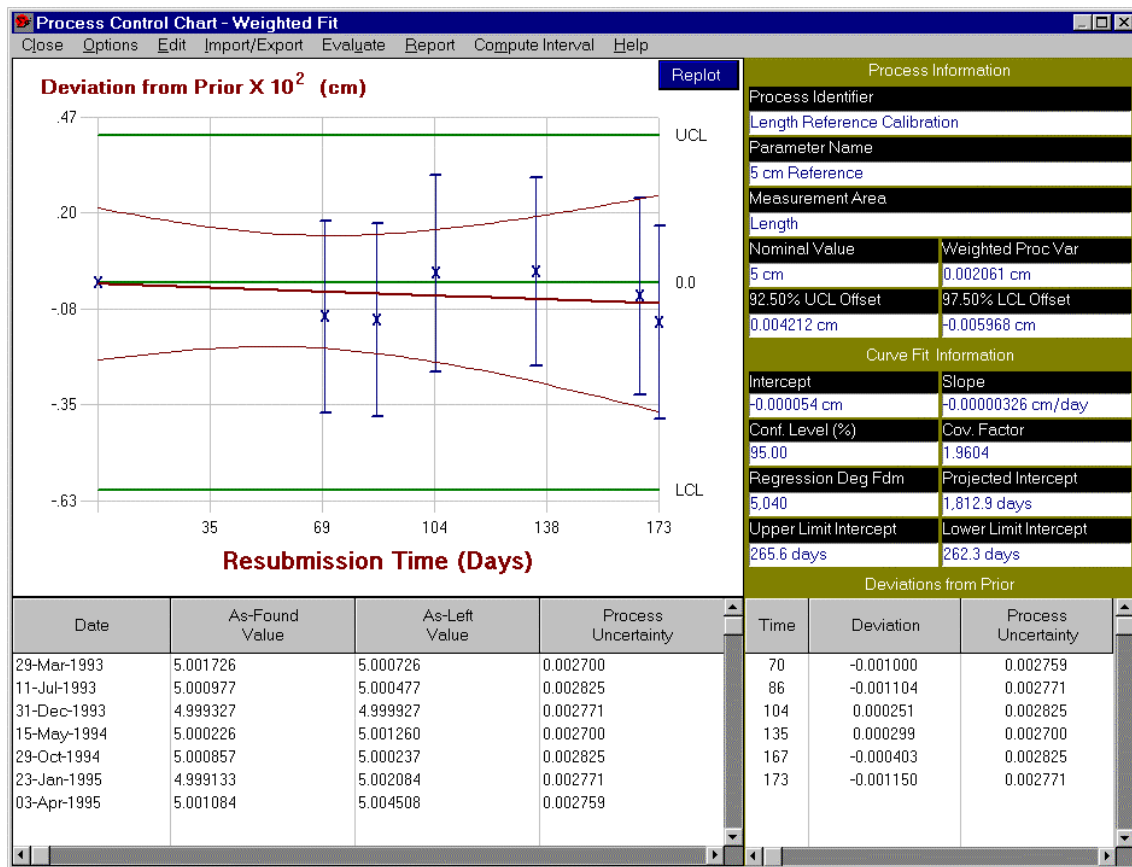
Figure 1. Control Chart for Deviations vs. Time. For the analysis shown, the chart indicates a slight negative drift in parameter bias as a function of time. Also shown is a standard deviation of 0.002061 cm due to random variations, labeled "Weighted Process Var." The measuring process uncertainties for each deviation are shown as ± one-sigma error bars. The screen was generated with SPCView [4].

---

[5] As indicated earlier, this should not be confused with the uncertainty due to random error in a measurement. This error contributes to the measurement process uncertainty shown for each plotted point. The random effects alluded to in the present discussion are experienced over a history of such measurements. They contribute to the uncertainty in projecting parameter value vs. time.

Estimating the Regression Coefficients

The regression coefficients are determined by minimizing a statistic called the *residual sum of squares* or *RSS*, given by[6]

$$RSS = \sum_{i=1}^{n} w_i \left(y_i - a - b t_i \right)^2 ,$$ (7)

where y_i is the observed difference between the ith as-found and as-left measurement, n is the number of observed differences, and t_i is the associated time elapsed between the ith difference. The variable w_i is the ith *weighting factor* or, simply, the ith "weight." The ith weighting factor is given by

$$w_i = \frac{c}{u_i^2} ,$$ (8)

where u_i is the measurement process uncertainty for the ith as-found/as-left difference, and

$$c = \frac{n}{\sum_{i=1}^{n} \left(1 / u_i \right)^2} .$$ (9)

Minimizing *RSS* produces two equations called the "normal equations"

$$a = \frac{1}{\Delta} \left[(\Sigma wt^2)(\Sigma wy) - (\Sigma wty)(\Sigma wt) \right],$$ (10)

and

$$b = \frac{1}{\Delta} \left[(\Sigma w)(\Sigma wty) - (\Sigma wt)(\Sigma wy) \right],$$ (11)

where

$$\Delta = (\Sigma w)(\Sigma wt^2) - (\Sigma wt)^2 .$$ (12)

In these expressions, Σw, Σwt, Σwty, Σwt^2 and Σwy are, respectively, summations from 1 to n of the weights; the product of the weights and the times; the product of the weights, times and observed differences; the product of the weights and times squared; and the product of the weights and the observed differences.

Practical Considerations

Since each y_i is actually a *difference* between an as-found value and a prior as-left value, we could justifiably say that, for each (y_i, t_i) pair entered into the analysis, we could also enter an additional pair $(y_i, 0)$, where the value of y_i would be an estimated quantity. Of course, since the elapsed time is zero for these additional pairs, the best estimate is $y_i = 0$ when $t_i = 0$.[7]

There is a definite benefit to doing this. To see this, imagine that we perform a regression analysis and estimate both a and b. Because of the nature of the data we might have available, the estimate for a may turn out to be considerably larger than what we know to be reasonable. That is, using the regression model $y(t) = a + bt$ may yield unrealistic values for $y(0)$.

If the $t = 0$ estimated pairs are included, the intercept coefficient a is forced toward a more reasonable value, namely,

---

[6] This is not to be confused with "root sum square" uncertainty combination. To try and keep the two separate, we will use the upper case RSS for residual sum of squares and the lower case rss for root sum square.

[7] It might be argued that two simultaneous measurements obtained at $t = 0$ with a given measurement process could yield different results due to measurement process error. This would suggest that setting $y_i = 0$ at $t = 0$ is not justified. This concern is accommodated by taking into account the uncertainty due to measurement error. In other words, we say that, at $t = 0$, we estimate the value of each y_i to be zero with uncertainty u_i.

zero.[8] In addition, the variance in *a* and the covariance term become smaller in magnitude. However, the variance in *b* becomes somewhat larger than what we would have if the estimated pairs were not included.

Projecting $y(t)$

Once we have obtained an estimate for the coefficient *b*, we can project values for $y(t)$, starting from a specific initial value y_0. This value may be a measurement result, a sampled mean value or a Bayesian estimate of the parameter bias at $t = 0$. The appropriate model is

$$y(t) = y_0 + bt , \qquad (13)$$

Note, that, although the coefficient *a* is not included, as in Eq. (5), we still need to take into account the covariance between *a* and *b*, since the uncertainty in the regression fit should not ignore the relationship between the uncertainties in the $t = 0$ values and the slope of the regression curve.

Letting u_{BOP} represent the uncertainty in y_0, we can write

$$u_{bias}^2 = u_{BOP}^2 + t^2 \, \text{var}(b) + 2t \, \text{cov}(a,b) , \qquad (14)$$

The uncertainty u_{BOP} may be estimated from a computed false accept risk, as will be done presently, or may be the result of a detailed uncertainty analysis [4-6]. If y_0 is obtained by Bayesian analysis, u_{BOP} becomes the standard deviation of the Bayesian estimate [7-14].

It should be remarked at this point that Eq. (14) is employed in estimating the bias uncertainty in a particular projected parameter value, starting from a specific initial value and initial uncertainty. This is not quite the same u_{bias} that is computed with Eq.(6). This uncertainty relates to computed values of $y(t)$ within the context of the regression analysis.

Estimating the Variances

If a value for $y(0)$ cannot be assumed, then we need to not only estimate the coefficient *a*, but also to compute its variance. This variance is given by

$$\text{var}(a) = \frac{s^2}{\Delta^2} \left[(\Sigma wt^2)^2 (\Sigma w^2) - 2(\Sigma wt^2)(\Sigma wt)(\Sigma w^2 t) + (\Sigma wt)^2 (\Sigma w^2 t^2) \right]$$
$$+ \frac{1}{\Delta^2} \left[(\Sigma wt^2)^2 (\Sigma w^2 u^2) - 2(\Sigma wt^2)(\Sigma wt)(\Sigma w^2 tu^2) + (\Sigma wt)^2 (\Sigma w^2 t^2 u^2) \right] , \qquad (15)$$

where the variable *u* in each sum represents the uncertainty u_i defined in the earlier definition of weighting factors, and where

$$s^2 = \frac{RSS}{n-2} . \qquad (16)$$

The variance in the coefficient *b* is written

$$\text{var}(b) = \frac{s^2}{\Delta^2} \left[(\Sigma w)^2 (\Sigma w^2 t^2) - 2(\Sigma w)(\Sigma wt)(\Sigma w^2 t) + (\Sigma wt)^2 (\Sigma w^2) \right]$$
$$+ \frac{1}{\Delta^2} \left[(\Sigma w)^2 (\Sigma w^2 t^2 u^2) - 2(\Sigma w)(\Sigma wt)(\Sigma w^2 tu^2) + (\Sigma wt)^2 (\Sigma w^2 u^2) \right] , \qquad (17)$$

and the covariance between *a* and *b* is

---

[8] This is actually a strength of the method, since it is expected that the average value for $y(0)$ will be zero. This does not mean, however, that the uncertainty in $y(0)$ will necessarily be zero.

$$\text{cov}(a,b) = \frac{s^2}{\Delta^2}\left\{(\Sigma wt^2)\left[(\Sigma w)(\Sigma w^2 t) - (\Sigma w^2)(\Sigma wt)\right] + (\Sigma wt)\left[(\Sigma wt)(\Sigma w^2 t) - (\Sigma w)(\Sigma w^2 t^2)\right]\right\}$$

$$+ \frac{1}{\Delta^2}\left\{(\Sigma wt^2)\left[(\Sigma w)(\Sigma w^2 tu^2) - (\Sigma w^2 u^2)(\Sigma wt)\right] + (\Sigma wt)\left[(\Sigma wt)(\Sigma w^2 tu^2) - (\Sigma w)(\Sigma w^2 t^2 u^2)\right]\right\}. \tag{18}$$

Special Case: Equal Process Uncertainties

If $u_1 = u_2 = \cdots = u_n = u$, then each $w_i = 1$, and the above expressions reduce to

$$\text{var}(a) = \frac{\Sigma t^2}{\Delta}(s^2 + u^2),$$

$$\text{var}(b) = \frac{n}{\Delta}(s^2 + u^2),$$

and

$$\text{cov}(a,b) = -\frac{\Sigma t}{\Delta}(s^2 + u^2)$$

where

$$\Delta = n(\Sigma t^2) - (\Sigma t)^2,$$

and s^2 is given in Eq. (16).

Projecting Intervals

Imagine that the parameter of interest is bounded by upper and lower tolerance limits $-L_1$ and L_2. If we knew the initial value, then we might suppose at first sight, that the approach to take in estimating a calibration interval would be to enter the initial value in Eq. (13) and solve for the time T required for y to cross either $-L_1$ or $+L_2$. If so, then, in cases where b is positive, we would have[9]

$$T = \frac{L_2 - y_0}{b}, \quad b > 0$$

while, for cases where b is negative, we would write

$$T = -\frac{L_1 + y_0}{b}, \quad b < 0.$$

This method, while conceptually palatable, is not recommended. Instead, we use two alternative approaches. In the first approach, the calibration interval is established as the time that corresponds to the point where the confidence that we are in-tolerance drops to some minimum acceptable level, given by $1 - \alpha$. The variable α is usually something like 0.05, 0.01, etc. In the second approach, the calibration interval is established as the time required for the uncertainty in the projected value of y to reach a maximum acceptable value.

Reliability Target Method

The probability or confidence that a parameter is in-tolerance is commonly referred to as *measurement reliability*. Accordingly, the approach for adjusting intervals to meet a given level of probability or confidence is labeled the *reliability target method*. We will now examine this method as applied to several alternative tolerancing options.

Two-Sided General Case: Asymmetric Tolerances

We consider a situation in which the upper and lower tolerance limits are not equal. We also suppose that the desired confidence for $y(t)$ being confined to values less than L_2 is $1 - \alpha$ and the desired confidence for $y(t)$ being restricted to values greater than $-L_1$ is $1 - \beta$.

We solve for T as the smallest of T_1 and T_2 using the expressions

---

[9] We assume that $-L_1 \leq y_0 \leq L_2$.

$$L_2 = y_0 + bT_2 + t_{\alpha,v}\sqrt{\mathrm{var}(y(T_2))}\,,$$

and

$$-L_1 = y_0 + bT_1 - t_{\beta,v}\sqrt{\mathrm{var}(y(T_1))}\,.$$

In these expressions, v represents the degrees of freedom of the variance estimates, and the variables $t_{\alpha,v}$ and $t_{\beta,v}$ are the t-statistics for confidence levels of $1 - \alpha$ and $1 - \beta$, respectively, with v degrees of freedom. The degrees of freedom v will be discussed later.

Solutions for T_1 and T_2 are obtained by iteration. A good method to use is the Newton-Raphson method. With this method, we first define a function F and its derivative F'. For the T_2 solution, these quantities are given by

$$F = bt + t_{\alpha,v}u_{bias} + y_0 - L_2\,,$$

and

$$F' = b + \frac{t_{\alpha,v}}{u_{bias}}\left[\mathrm{cov}(a,b) + t\,\mathrm{var}(a)\right]\,,$$

where u_{bias} is given as

$$u_{bias} = \sqrt{u_{BOP}^2 + 2t\,\mathrm{cov}(a,b) + t^2\,\mathrm{var}(b)}\,. \qquad (19)$$

We next estimate a starting value for t and solve for the value of t that makes the magnitude of the ratio F/F' smaller than some desired level of precision ε. The iteration algorithm is

```
Set t = t₀
Compute F/F′
Do until Abs(F/F′) < ε
    t = t - F/F′
    Compute F/F′
Loop
```

Following completion of the loop, we set $T_2 = t$. The variable T_1 is solved with the same algorithm, except that F and F' are now given by

$$F = bt - t_{\beta,v}u_{bias} + y_0 + L_1\,,$$

and

$$F' = b - \frac{t_{\beta,v}}{u_{bias}}\left[\mathrm{cov}(a,b) + t\,\mathrm{var}(a)\right]\,,$$

where u_{bias} is computed using Eq. (19).

Two-Sided Symmetric Tolerances

In this case, we have $L_1 = L_2 = L$, and, assuming equal upper and lower confidence levels, we have $\alpha_1 = \alpha_2 = \alpha/2$. As before, we solve for T as the smallest of T_1 and T_2. These variables are again solved for iteratively using the algorithm described above.

For the T_2 solution, the functions F and F' are given by

$$F = bt + t_{\alpha/2,v}u_{bias} + y_0 - L\,,$$

and

$$F' = b + \frac{t_{\alpha/2,v}}{u_{bias}}\left[\mathrm{cov}(a,b) + t\,\mathrm{var}(a)\right]\,.$$

For the T_1 solution, we have

$$F(t) = bt - t_{\alpha/2,v}u_{bias} + y_0 + L\,,$$

and

$$F' = b - \frac{t_{\alpha/2,v}}{u_{bias}} \left[\text{cov}(a,b) + t \, \text{var}(a) \right].$$

For both solutions, the quantity u_{bias} is again given by Eq. (19).

Two interval analyses using the regressions results of Figure 1 are shown in Figures 2 and 3. Figure 2 projects an interval for an initial value of 0.001 cm, while Figure 3 projects an interval for an initial value of 0.000 cm. For the analyses shown, the zero initial value case yields the longer of the two intervals.
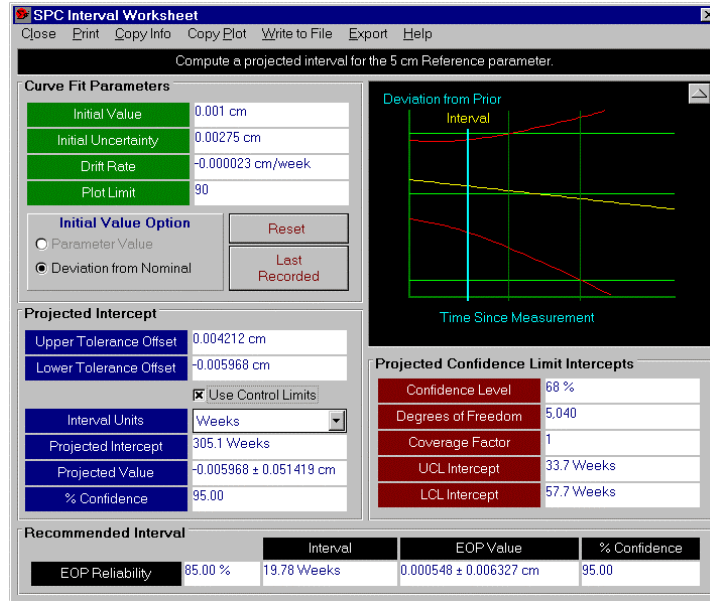
Figure 2. Interval Analysis – Known Uncertainty and Nonzero Initial Value. The projected interval of approximately 20 weeks reflects an initial value of 0.001 cm, an initial uncertainty u_{BOP} of 0.00275 cm, and corresponds to a confidence level of 85%. The projected bias at the end of the interval is 0.000548 cm ±0.006327 cm. Note that the upper and lower control limits are asymmetric. The analysis is based on the regression results shown in Figure 1. The graphic is a screen shot of the SPC Interval Worksheet of SPCView [4].

As a side note; for this particular example, it would seem prudent to center spec the calibrated parameter at each calibration. Other examples may yield different conclusions. In some instances, the conclusion is counter-intuitive. For instance, we may have a parameter whose value exhibits a negative drift rate, leading us to suspect that adjusting the parameter value upward at calibration will yield an extended interval. However, since the interval is keyed to a target confidence level, rather than an intercept time, this does not always happen. In some cases, such compensating adjustments actually lower the in-tolerance probability.

General Single-Sided Upper Limit Case

For cases where the parameter of interest has only a single "not to exceed" tolerance limit *L*, we attempt to set a calibration interval that corresponds to a minimum acceptable confidence level 1 - α that the parameter's value will be less than or equal to *L*. The relevant expression is

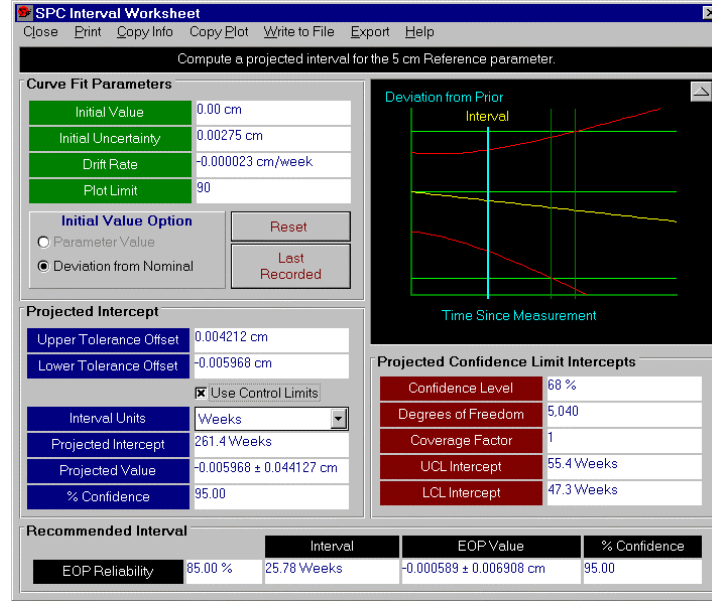$$L = y_0 + bT + t_{\alpha,v} \sqrt{\text{var}(y(T))} ,$$

The solution for the interval *T* is arrived at by iteration using the algorithm given for the asymmetric two-sided case with

$$F(t) = bt + t_{\alpha,v} u_{bias} + y_0 - L ,$$

and

$$F' = b + \frac{t_{\alpha,v}}{u_{bias}}\left[\mathrm{cov}(a,b) + t\,\mathrm{var}(a)\right],$$

where u_{bias} is again given by Eq. (19).

Figure 3. Interval Analysis – Known Initial Uncertainty and Zero Initial Value. For the analysis shown, we can extend the interval of Figure 2 by about six weeks if we adjust the initial value to zero.

General Single-Sided Lower Limit Case

In this case, we attempt to determine a calibration interval that corresponds to a minimum acceptable confidence level 1 - α that the parameter's deviation from y_0 will be greater than or equal to L. The relevant expression is

$$L = y_0 + bT - t_{\alpha,v}\sqrt{\mathrm{var}(y(T))},$$

The solution for the interval T employs the same algorithm as the single-sided upper case with

$$F(t) = bt - t_{\alpha/2,v}u_{bias} + y_0 + L,$$

and

$$F' = b - \frac{t_{\alpha/2,v}}{u_{bias}}\left[\mathrm{cov}(a,b) + t\,\mathrm{var}(a)\right].$$

Single-Sided Cases with Known Initial Value and Uncertainty

In this case, we have

$$L_\pm = y_0 + bT \pm t_{\alpha,v}\left[u_{BOP}^2 + T^2\,\mathrm{var}(b)\right],$$

where the +(-) solution applies to cases with an upper (lower) tolerance limit. The solution is

$$T = -c_\pm + \sqrt{c_\pm^2 + d_\pm},$$

where

$$c_\pm = \pm\frac{b}{2t_{\alpha,v}\,\mathrm{var}(b)},$$

and

$$d_\pm = \pm\frac{L - y_0 \pm t_{\alpha,v}u_{BOP}^2}{t_{\alpha,v}\,\mathrm{var}(b)}.$$

Uncertainty Target Method

We return to the assertion that the uncertainty in a projected parameter value increases with time elapsed since measurement. The approach for controlling this uncertainty to a maximum allowable value is labeled the *uncertainty target method*.

With this method, the tolerance of the parameter of interest is not a factor. Instead, we solve for the time T that it takes u_{bias} to grow from an initial value u_{BOP} to some maximum allowable target value u^*.
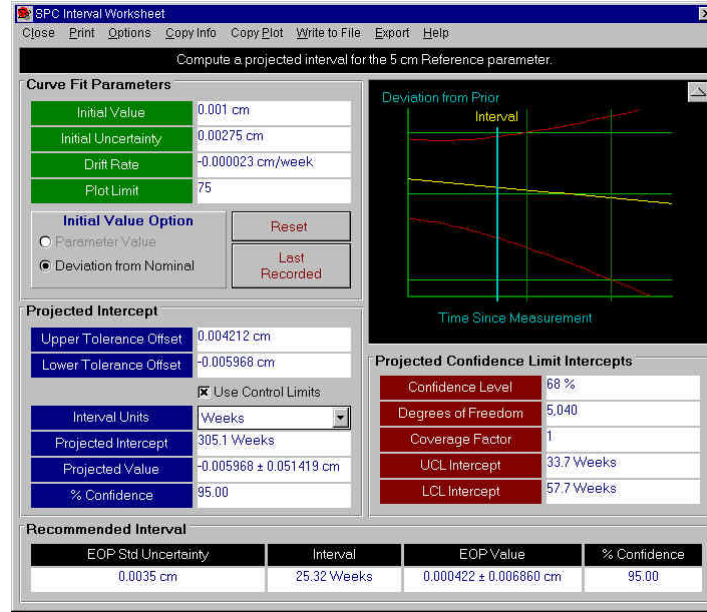
Figure 4. Interval Analysis to Meet an Uncertainty Target – Known Uncertainty and Nonzero Initial Value.
The example shown reflects an initial uncertainty u_{BOP} of 0.00275 cm and a drift rate of –0.000023 cm/week. The time required for the bias uncertainty to reach the maximum allowable value of 0.0035 cm is approximately 25 weeks. The projected bias at the end of that interval is 0.000422 cm ±0.00687 cm.

The relevant expression is

$$(u^*)^2 = u_{BOP}^2 + T^2 \operatorname{var}(b) + 2T \operatorname{cov}(a,b) \ .$$

Solving for the T gives

$$T = -c + \sqrt{c^2 - d} \ ,$$

where

$$c = \frac{\operatorname{cov}(a,b)}{\operatorname{var}(b)}$$

and

$$d = \frac{u_{BOP}^2 - (u^*)^2}{\operatorname{var}(b)} \ .$$

Computing the Degrees of Freedom

The degrees of freedom ν used to determine the appropriate t-statistics in this paper is obtained using the Welch-Satterthwaite relation [6]

$$\nu = \frac{u_{total}^4}{\sum_{i=1}^{k} \frac{u_i^4}{\nu_i}} \ ,$$

where k is the number of components of u_{total}. If each uncertainty component has associated with it a sensitivity coefficient κ_i, then the Welch-Satterthwaite relation becomes

$$\nu = \frac{u_{total}^4}{\displaystyle\sum_{i=1}^{k} \frac{(\kappa_i u_i)^4}{\nu_i}} \, .$$

This relation will now be employed to compute the degrees of freedom for u_{bias}.

To obtain the degrees of freedom, we need to develop an expression for u_{total} that expresses the terms var(a), var(b) and cov(a,b) of Eq. (6) in a form that is conducive to using the Welch-Satterthwaite relation. To do this, we write

$$\text{var}(a) = \alpha_0^2 s^2 + \sum_{i=1}^{n} \alpha_i^2 u_i^2 \,,$$

$$\text{var}(b) = \beta_0^2 s^2 + \sum_{i=1}^{n} \beta_i^2 u_i^2 \,,$$

and

$$\text{cov}(a,b) = \lambda_0^2 s^2 + \sum_{i=1}^{n} \lambda_i^2 u_i^2 \,,$$

where, from Eqs. (15), (17) and (18), we have

$$\alpha_0^2 = \frac{1}{\Delta^2}\left[(\Sigma wt^2)^2(\Sigma w^2) - 2(\Sigma wt^2)(\Sigma wt)(\Sigma w^2 t) + (\Sigma wt)^2(\Sigma w^2 t^2)\right],$$

$$\alpha_i^2 = \frac{1}{\Delta^2}\left[(\Sigma wt^2) - (\Sigma wt)t_i\right]^2 w_i^2, \quad i = 1, 2, \cdots, n \,,$$

$$\beta_0^2 = \frac{1}{\Delta^2}\left[(\Sigma w)^2(\Sigma w^2 t^2) - 2(\Sigma w)(\Sigma wt)(\Sigma w^2 t) + (\Sigma wt)^2(\Sigma w^2)\right],$$

$$\beta_i^2 = \frac{1}{\Delta^2}\left[(\Sigma wt) - (\Sigma w)t_i\right]^2 w_i^2, \quad i = 1, 2, \cdots, n \,,$$

$$\lambda_0^2 = \frac{1}{\Delta^2}\left\{(\Sigma wt^2)\left[(\Sigma w)(\Sigma w^2 t) - (\Sigma w^2)(\Sigma wt)\right] + (\Sigma wt)\left[(\Sigma wt)(\Sigma w^2 t) - (\Sigma w)(\Sigma w^2 t^2)\right]\right\},$$

and

$$\lambda_i^2 = -\frac{1}{\Delta^2}\left\{(\Sigma wt^2)(\Sigma wt) - \left[(\Sigma wt^2)(\Sigma w) + (\Sigma wt)^2\right]t_i + (\Sigma wt)(\Sigma w)t_i^2\right\} w_i^2 \quad, i = 1, 2, \cdots, n \,.$$

We now return Eq. (6) and write

$$u_{bias}^2 = \text{var}(a) + t^2 \, \text{var}(b) + 2t \, \text{cov}(a,b)$$

$$= h_0^2(t)s^2 + \sum_{i=1}^{n} h_i^2(t)u_i^2,$$

where

$$h_i^2(t) = \alpha_i^2 + 2\lambda_i^2 t + \beta_i^2 t^2 \quad i = 0, 1, 2 \cdots, n \,.$$

Using this formalism, the degrees of freedom ν_{bias} is obtained from

$$v_{bias} = \frac{u_{bias}^4}{\dfrac{[h_0(t)s]^4}{n-2} + \displaystyle\sum_{i=1}^{n} \frac{[h_i(t)u_i]^4}{v_i}} \quad .$$

Attributes Data Analysis

In managing calibration and test equipment inventories, we rarely have the as-found and as-left variables data we have been working with to this point. Instead, we are usually forced to work with as-found "in-tolerance" or "out-of-tolerance" records. Data of this sort are referred to as *attributes data*.

Attributes data typically consist of a service date coupled with an as-found condition or "condition received." The condition received variable may take on several possible values including "in-tolerance" (success), "out-of-tolerance" (failure), "damaged," "inoperative," etc. Controlling uncertainty growth through the analysis of attributes data is covered in detail if References 1 and 2. The following is intended to serve as a thumbnail view of the material in those documents.

The Resubmission Time Series

The successive dates between service actions on an item of equipment constitute an observed interval or *resubmission time*. The resubmission time may be reset following each service or may be allowed to run until the occurrence of a recorded adjustment, corrective action or other "renewal." Depending on the method of analysis and assumptions about equipment renewal, a given condition received may be treated as a success or failure, may be ignored, or may be used to simply reset the clock with regard to resubmission time.

From condition received data, a set of observed in-tolerance probabilities or percents in-tolerance is compiled that couples the observed probability with a corresponding resubmission time. Such a compilation is called a *time series*. Since the condition received observations are often taken at the end of a scheduled calibration interval, the in-tolerance probabilities are commonly referred to as "end-of-period percents in-tolerance," "% EOP" or, simply, "EOP." A typical time series is shown in Table 1. In Table 1, service actions are grouped by resubmission time and observed reliabilities are computed for each resubmission time grouping.[10] In compiling such a time series, observed data may be portrayed for an individual item or for a homogeneous grouping of items.

TABLE 1
Example Out-of-Tolerance Time Series[11]

| Weeks Between Calibrations t	Number Calibrations Recorded n(t)	Number In-Tolerances Observed g(t)	Observed Measurement Reliability R(t)
2-4	4	4	1.0000
5-7	6	5	0.83333
8-10	14	9	0.6429
11-13	13	8	0.6154
19-21	22	12	0.5455
26-28	49	20	0.4082
37-40	18	9	0.5000
48-51	6	2	0.3333

---

[10] At present, research is planned to evaluate using ungrouped attributes data in analyzing uncertainty growth. The results of this research will be reported in a future paper.

[11] Taken from Table C1 of Reference 2.

Measurement Reliability Modeling

The transition of an equipment parameter from an in-tolerance state to an out-of-tolerance state is a random event. Accordingly, the process by which this transition occurs is called a *stochastic process*. To analyze the observed time series, a mathematical model is assumed for the stochastic process. The model is a mathematical function, characterized by coefficients. The functional form is specified while the coefficients are estimated on the basis of the observed time series. The problem of determining the probability law for the stochastic process thus becomes the problem of selecting the correct functional form for the time series and estimating its coefficients.

Since an out-of-tolerance condition is analogous to a "measurement accuracy failure," the in-tolerance probability is called the *measurement reliability*. The mathematical model for the time series is called the *reliability model*.
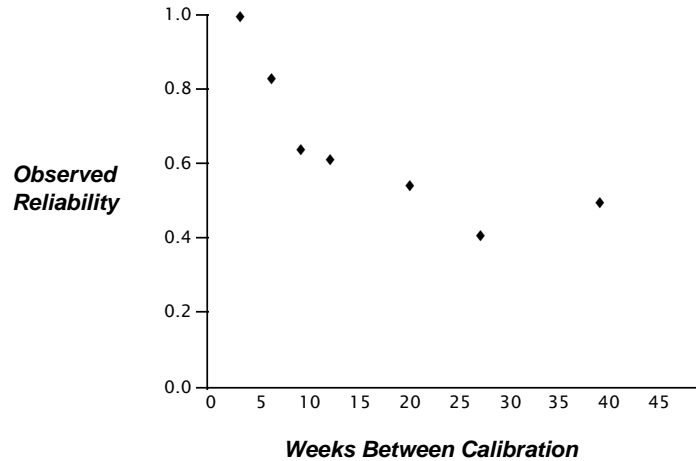
Figure 5. Hypothetical Observed Time Series. The observed measurement reliabilities for the time series tabulated in Table 1.

The method used to estimate the coefficients of a reliability model involves choosing a functional form which yields meaningful predictions of measurement reliability as a function of time. By its nature, the function cannot precisely predict the times at which transitions to out-of-tolerance occur. Instead, it predicts measurement reliability expectation values, given the times elapsed since calibration. Thus the analysis attempts to determine a predictor $\hat{R}(t,\hat{\theta}) = R(t) + \varepsilon$, where the random variable ε satisfies $E(\varepsilon) = 0$. It can be shown that the method of maximum likelihood estimation provides consistent reliability model coefficient estimates for such predictors [15].
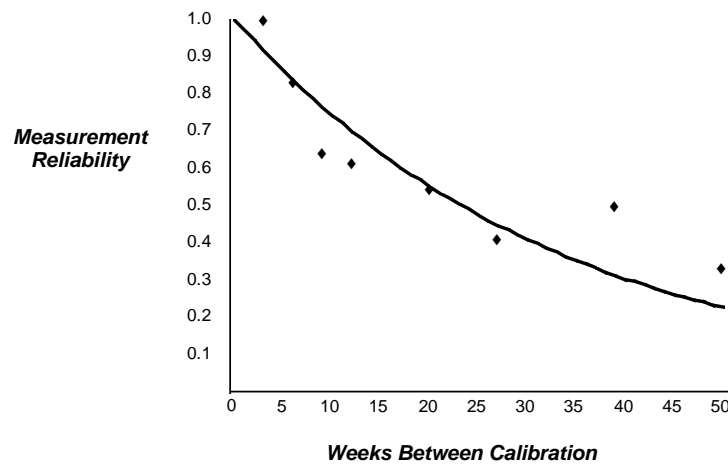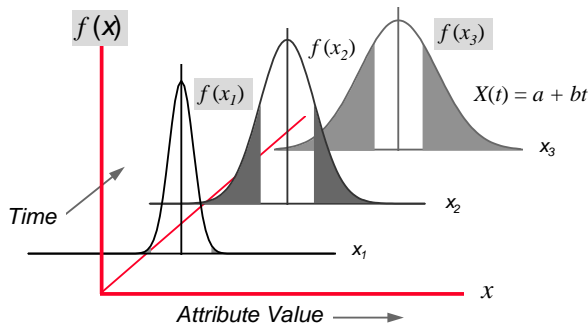

Figure 6. Out-of-Tolerance Stochastic Process Model. The stochastic process underlying the time series is modeled by an exponential function of the form $R(t) = R_0 e^{-\lambda t}$.

Whether the aim is to ensure measurement integrity for periodically calibrated equipment or to design equipment to tolerate extended periods between calibration, the uncertainty growth stochastic process is described in terms of mathematical models, characterized by two features: (1) a functional form, and (2) a set of coefficients. Figure 6 models the time series of Table 1 with an exponential reliability model $R(t) = R_0 e^{-\lambda t}$ characterized by the coefficients $R_0 = 1$ and $\lambda = 0.03$. Determination as to which mathematical form is appropriate for a given stochastic process and what values are to be assigned the coefficients is documented in the literature [1, 2].
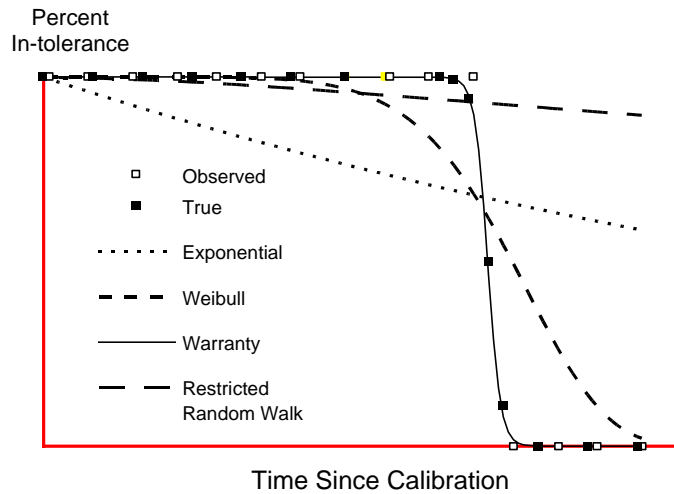
Estimating Uncertainty Growth

Our knowledge of the values of the measurable attributes of a calibrated item begins to fade from the time the item is calibrated. This loss of knowledge of the values of attributes over time is called uncertainty growth. For many attributes, there is a point where uncertainty growth reaches an unacceptable level, creating a need for recalibration. Determining the time from the date of calibration required for an attribute's uncertainty to grow to an unacceptable level is the principal endeavor of calibration interval analysis.
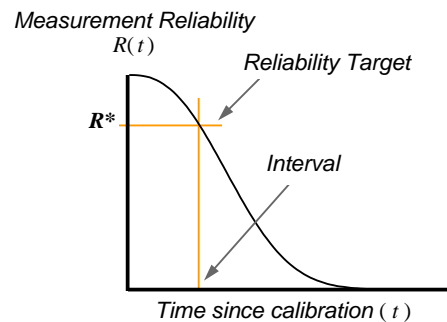
An unacceptable level of uncertainty corresponds to an unacceptable out-of-tolerance probability and a higher expected incidence of out-of-tolerance conditions. For analysis purposes, an out-of-tolerance condition is regarded as a kind of "failure," similar to a component or other functional failure. However, unlike functional failures that are obvious to equipment users and operators, out-of-tolerance failures usually go undetected during use. The detection of such failures occurs at calibration, provided of course that calibration uncertainties are sufficiently small.

Figure 7. Measurement Uncertainty Growth Mechanisms. Several mathematical functions have been found applicable for modeling measurement uncertainty growth over time.

Figure 8. Measurement Uncertainty Growth. Uncertainty growth over time for a typical measurement attribute. The sequence shows statistical distributions at three different times. The uncertainty growth is reflected in the spreads in the curves. The out-of-tolerance probabilities at the times shown are represented by the shaded areas under the curves (the total area of each curve is equal to unity.). As can be seen, the growth in uncertainty over time corresponds to a growth in out-of-tolerance probability over time.

Figure 9. Measurement Reliability vs. Time. The picture of uncertainty growth in Figure 8 shows that the in-tolerance probability, or measurement reliability, decreases with time since calibration. Plotting this quantity vs. time suggests that measurement reliability can be modeled by a time-varying function. Once this function is determined, the uncertainty in the bias of a parameter may be computed as a function of time.

Several uncertainty growth mechanisms have been observed in practice. The most versatile of these are described in References 1 and 2 and have been incorporated in commercially available software [5, 16, 17]. Five mechanisms are shown in Figure 6 that illustrate the differences in some of the applicable mechanisms. From the figure, it is evident that reliability models are not always interchangeable.

Computing Bias Uncertainty

As is shown in the previous section, the uncertainty in the bias of a parameter value at a given time elapsed since measurement can be linked to the in-tolerance probability at that time. The relationship can be written

$$R(t) = \int_{-L_1}^{L_2} f[(x, u(t)]dx \,,$$

where

$$
\begin{aligned}
x = \quad & \text{parameter bias or deviation from nominal} \\
& \text{or deviation from expected value} \\
u(t) = \quad & \text{the standard uncertainty in x at time } t \\
f = \quad & \text{the probability density function for } x \\
-L_1 = \quad & \text{the lower tolerance limit for } x \\
L_2 = \quad & \text{the upper tolerance limit for } x
\end{aligned}
$$

Of course, in actual practice, we would need to employ a reliability model for $R(t)$, for which we have a set of estimated coefficients. Accordingly, we would have

$$\hat{R}(t, \hat{\theta}) = \int_{-L_1}^{L_2} f[(x, u(t)]dx \,.$$

Once the reliability model is selected and modeled and a form can be decided on for $f[x, u(t)]$, what remains is to invert the above expression to solve for $u(t)$. The approach will be illustrated for normally distributed parameter biases.

Using the Normal Distribution

Most uncertainty analysis methods and techniques are built on the assumption that the errors or biases, whose uncertainties we are attempting to estimate, are normally distributed [6, 18]. Given this assumption, the probability density function can be written

$$f[x, u(t)] = \frac{1}{\sqrt{2\pi}u(t)} e^{-x^2/2u^2(t)} \,,$$

where it is assumed that the mean value for the error or parameter bias of interest is zero. Defining the function

$$\Phi(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{w} e^{-\zeta^2/2}d\zeta \,,$$

we have, after a little rearranging,

$$\hat{R}(t, \hat{\theta}) = \frac{1}{\sqrt{2\pi}} \int_{-L_1/u(t)}^{L_2/u(t)} e^{-x^2/2} \, dx$$

$$= \Phi\left[\frac{L_1}{u(t)}\right] + \Phi\left[\frac{L_2}{u(t)}\right] - 1.$$

The uncertainty $u(t)$ can be solved for numerically from this expression. The Newton-Raphson method described earlier may be employed for this purpose. In this case, we actually solve for a variable $\lambda = u(t)^{-1}$. For the present solution, we have

$$F = \Phi\left[\lambda L_1\right] + \Phi\left[\lambda L_2\right] - 1 - \hat{R}(t, \hat{\theta}) \,,$$

and

$$F' = \frac{L_1}{\sqrt{2\pi}} e^{-(\lambda L_1)^2/2} + \frac{L_2}{\sqrt{2\pi}} e^{-(\lambda L_2)^2/2} \,.$$

The algorithm is just

```
Set λ = 1 / u₀
Compute F/F′
Do until Abs(F/F′) < ε
        λ = λ - F/F′
        Compute F/F′
Loop
Set u = 1 / λ
```

Special Case: $L_1 = L_2 = L$. In the event that the upper and lower tolerances are equal, we have

$$\hat{R}(t,\hat{\theta}) = 2\Phi\left[\frac{L}{u(t)}\right] - 1 \; ,$$

and

$$u(t) = \frac{L}{\Phi^{-1}\left[\dfrac{1+\hat{R}(t,\hat{\theta})}{2}\right]} \; .$$

The function Φ^{-1} is the inverse normal function found in statistics texts and popular spreadsheet programs. For example, in Microsoft Excel, the function is called NORMSINV.

Other Distributions

While extremely useful for uncertainty analysis, the normal distribution is not applicable for certain sources of error nor is it usable in the absence of certain information. These considerations are covered Reference 18.

BOP Uncertainty

If values of $y(t)$ are computed from Eq. (5), then $u_{BOP} = \text{var}(a)$, where the variance is given in Eq. (15). If, instead, $y(t)$ is a projection from an initial value y_0, as in Eq. (13), then u_{BOP} is estimated as the initial uncertainty in y_0. There are two basic method for obtaining such an estimate; parameter normalization and tolerance testing.

Parameter Normalization – Variables Data

If the result of a test or calibration is an assignment of an estimated value, either through a physical parameter adjustment or through the publication of a value or correction, then u_{BOP} is essentially equal to the uncertainty of the test or calibration process.[12] In some cases, it may be necessary to include an additional uncertainty associated with making a physical adjustment. This is especially so in cases where physical adjustments may produce unknown spontaneous changes in parameter value [19].

For some parameters, it may also be advisable to include an uncertainty contribution due to errors induced by the delivery of a tested or calibrated device to the user. This "shipping stress" uncertainty is relevant in cases where the process of shipping and handling involves stresses that differ appreciably from those encountered during use.

Tolerance Testing – Attributes Data

Certain tests or calibrations may consist only of a check to see whether the subject parameter can be said to be in-tolerance. Ordinarily, a parameter is said to be in-tolerance if the measurement result of testing or calibration is contained within the parameter's tolerance limits. If Bayesian methods are applied, the in- or out-of-tolerance decision can often be refined.

---

[12] Under certain conditions, it may be possible to apply Bayesian methods of analysis to the measurement result of a test or calibration [1, 7-10]. Such methods refine the estimated value and reduce its uncertainty. Bayesian methods may also be employed in tolerance testing.

With tolerance testing, the u_{BOP} term can be obtained from an estimate of the parameter's BOP in-tolerance probability. This probability is equal to the *false accept risk* associated with the test or calibration. This risk, defined as the probability that an accepted parameter is out-of-tolerance, is a function of the uncertainty of the measurement process, the tolerance specifications of the subject parameter, and this parameter's in-tolerance probability as received for testing or calibration [17, 20, 21].

If the false accept risk is represented by the variable p_{fa}, then the BOP in-tolerance probability is given by[13]

$$p_0 = 1 - p_{fa}.$$

If the distribution of BOP parameter biases is known, the determination of u_{BOP} immediately follows. For example, if these biases are normally distributed, and the subject parameter's spec is stated as symmetric two-sided limits $\pm L$, then u_{BOP} is given by

$$u_{BOP} = \frac{L}{\Phi^{-1}\left(\frac{1 + p_0}{2}\right)}$$

$$= \frac{L}{\Phi^{-1}\left(1 - p_{fa}/2\right)},$$

where Φ^{-1} is the inverse normal distribution function.

Of course, other distributions are possible. Reference 18 describes several distributions that are useful for computing bias uncertainty and discusses their applicability. In this reference, the applicability (or lack thereof) of the uniform distribution is given special attention.

REFERENCES

[1] Metrology — Calibration and Measurement Processes Guidelines, NASA Reference Publication 1342, June 1994.

[2] Recommended Practice RP-1, Establishment and Adjustment of Calibration Intervals, NCSL International, Boulder, CO, January 1996.

[3] Castrup, H., "Uncertainty Growth Estimation in Uncertainty Analyzer," Integrated Sciences Group, http://www.isgmax.com, August 23, 2000.

[4] SPCView, ©1995 – 2001, Integrated Sciences Group, Bakersfield, CA, all rights reserved.

[5] UncertaintyAnalyzer, ©1994-1997, Integrated Sciences Group, Bakersfield, CA, all rights reserved.

[6] ISO, Guide to the Expression of Uncertainty in Measurement, Geneva, Switzerland, 1995.

[7] Castrup, H., "Intercomparison of Standards: General Case," SAI Comsystems, D.O. 4M03, Dept. of the Navy Contract N00123-83-D-0015, 16 March 1984.

[8] Jackson, D.H., "A Derivation of Analytical Methods to be Used in a Manometer Audit System Providing Tolerance Testing and Built-In Test," SAIC/MED-TR-830016-4M112/005-01, Dept. of the Navy, NWS Seal Beach, 8 October 1985.

[9] Jackson, D.H., "Analytical Methods to be Used in the Computer Software for the Manometer Audit System," SAIC/MED-TR-830016-4M112/006-01, Dept. of the Navy, NWS Seal Beach, 8 October 1985.

[10] Castrup, H., "Analytical Metrology SPC Methods for ATE Implementation," Proc. NCSL 1991 Workshop and Symposium, Albuquerque, July 1991.

---

[13] In obtaining p_0 from p_{fa}, false accept risk should be calculated as the probability of finding an out-of-tolerance parameter in the population of accepted parameters. This option is contrasted with the customary definition in which false accept risk is calculated as the probability of erroneously accepting an out-of-tolerance parameter. The latter definition yields a smaller value of p_{fa} than what is obtained under the former definition. Both definitions are described in the literature [20, 21] and are implemented in the software product AccuracyRatio [17].

[11] Cousins, R., "Why Isn't Every Physicist a Bayesian?" *Am. J. Phys.*, Vol 63, No. 5, May 1995.

[12] Kacker, R., "A Method to Quantify Uncertainty Due to Bias in Chemical Analyses," Proc. Measurement Science Conference, Anaheim, January 2000.

[13] Kacker, R., "An Interpretation of the Guide to the Expression of Uncertainty in Measurement," Proc. NCSL 2000 Workshop & Symposium, Toronto, July 2000.

[14] Kacker, R., "Towards a Simpler Bayesian Guide to the Expression of Uncertainty in Measurement," Proc. Measurement Science Conference, Anaheim, January 2001.

[15] Wold, H., "Forecasting by the Chain Principle," *Time Series Analysis*, ed. by M. Rosenblatt, pp 475-477, John Wiley & Sons, Inc., New York, 1963.

[16] IntervalMAX, © 1995-2000, Integrated Software Associates, Distributed by Integrated Sciences Group, Bakersfield, CA 93306.

[17] AccuracyRatio, ©1992-2001, Integrated Sciences Group, Bakersfield, CA, all rights reserved.

[18] Castrup, H., "Distributions for Uncertainty Analysis," Proc. IDW 2001 Workshop, Knoxville, May 2001.

[19] Castrup, H. and Jackson, D., "Uncertainty Propagation in Calibration Support Hierarchies," Presented at the NCSLI 2001 Workshop and Symposium, Washington D.C., July 2001.

[20] Castrup, H., "Risk-Based Control Limits," Proc. 2001 Measurement Science Conference, Anaheim, January 2001.

[21] Castrup, H., "Test and Calibration Metrics," Integrated Sciences Group, http://www.isgmax.com, Bakersfield, February 2001.